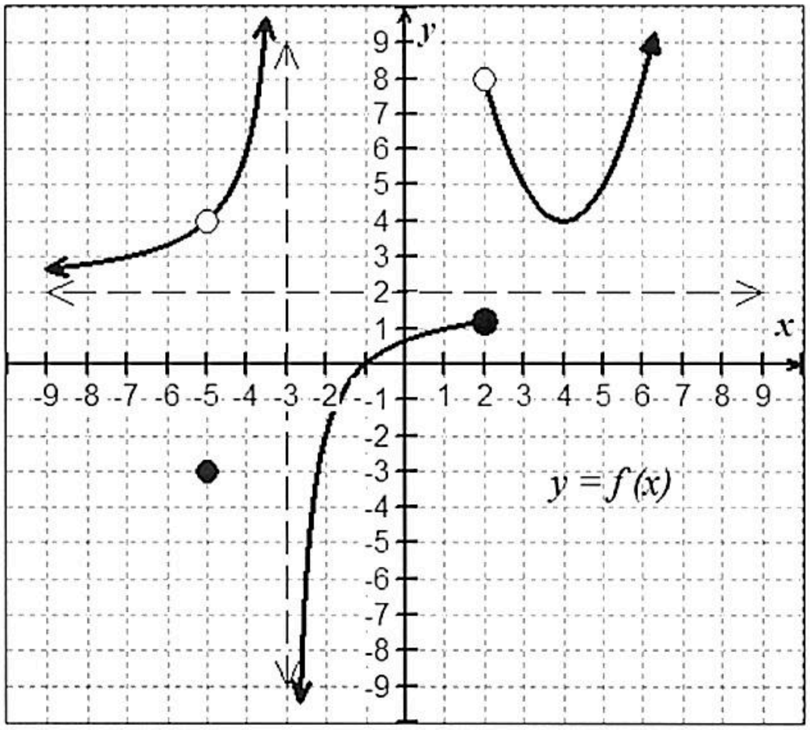
Understanding Limits Graphically and Numerically

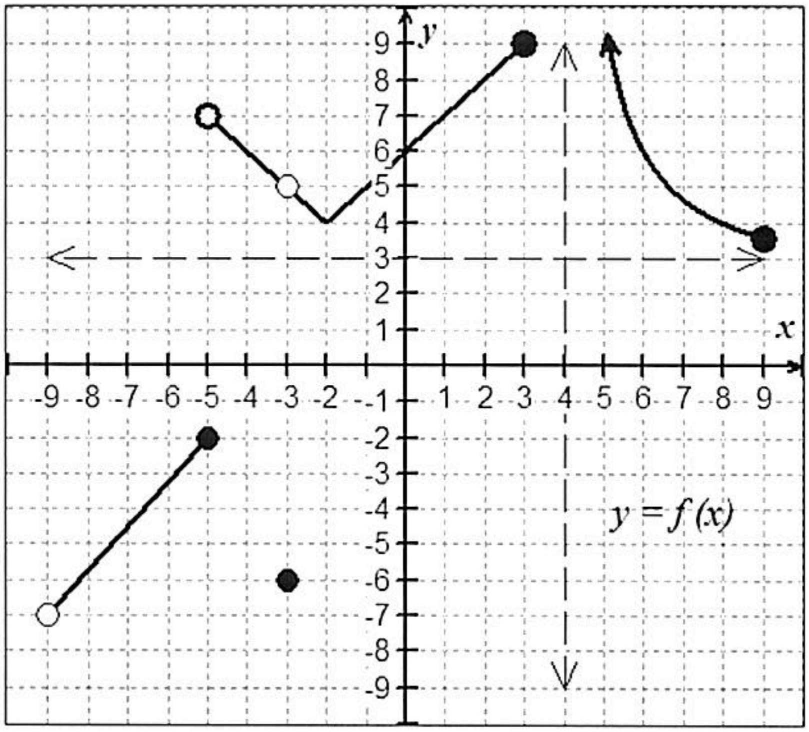
Consider the graph of the function $f(x)$, graphed below:



Using the graph, find the value of each of the following limits. If a limit does not exist, explain why.

A. $\lim_{x\to -3^-} f(x)$	B. $\lim_{x\to -5} f(x)$	C. $\lim_{x\to 4} f(x)$
D.) $\lim_{x\to 2^+} f(x)$	E.) $\lim_{x\to 2^-} f(x)$	F.) $\lim_{x\to 2} f(x)$
G.) $\lim_{x\to -1} f(x)$	H.) $\lim_{x\to -\infty} f(x)$	I.) $\lim_{x\to \infty} f(x)$

Now you give it a try. Consider the graph shown below to find the value of each of the following limits. If a limit does not exist, explain why.



A. $\lim_{x \to -5^+} f(x)$	B. $\lim_{x \to -2} f(x)$	C. $\lim_{x \to -3} f(x)$
D.) $\lim_{x \to 3^+} f(x)$	E.) $\lim_{x \to 3^-} f(x)$	F.) $\lim_{x \to -5^-} f(x)$
G.) $\lim_{x \to 0} f(x)$	H.) $\lim_{x \to -9} f(x)$	I.) $\lim_{x \to 4^+} f(x)$

Limits are the "backbone" of understanding that connect algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific x-value. Recall from Pre-Calculus that you evaluated three types of limits. Complete the table below:

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	PROPER NOTATION	VERBALLY:
Right-hand limit		
Left-hand limit		
General limit		

Consider the function shown below.
Say you want to find $\lim\limits_{x \to 4^+} f(x)$, the positive sign in the limit notation indicates a right-hand limit.
If you think of the function as a highway and imagine you are traveling along the graph of $f(x)$ toward $x = 4$ FROM THE RIGHT, NOT TO THE RIGHT, and you stop at the vertical line $x = 4$, the y-value where you stop is 3. Therefore, $\lim\limits_{x \to 4^+} f(x) = 3$.

You will use this graph to explore the limits for the problems on the next page.

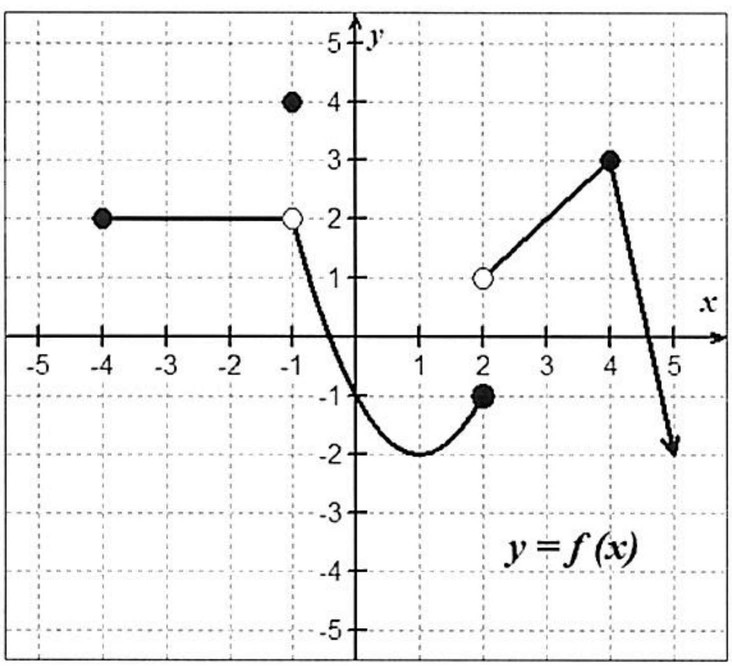


Figure 1-1

EX #1: Use Figure 1-1 to find the function values and evaluate each of the following limits:

1. $f(2)$	2. $f(-1)$
3. $\lim\limits_{x \to 4^-} f(x)$	4. $\lim\limits_{x \to 2^+} f(x)$
5. $\lim\limits_{x \to 2^-} f(x)$	6. $\lim\limits_{x \to -1^+} f(x)$
7. $\lim\limits_{x \to -1^-} f(x)$	8. $\lim\limits_{x \to -4^+} f(x)$
9. $\lim\limits_{x \to -4^-} f(x)$	10. $\lim\limits_{x \to -1} f(x)$
11. $\lim\limits_{x \to 2} f(x)$	12. $\lim\limits_{x \to 5} f(x)$
13. $\lim\limits_{x \to 0} f(x)$	14. $\lim\limits_{x \to 1} f(x)$

EX #2: Think about this!

If we think of the function as a highway, then the point at $(2, -1)$ could be considered the end of the road, while the point at $(-1, 2)$ is more like a "pothole." How would you describe the points located at

$(2, 1)$: ______________________________

$(4, 3)$: ______________________________

Hopefully, this analogy gives you a visual reference for understanding limits from a graphical approach. Let's get a little more formal with our definition now.

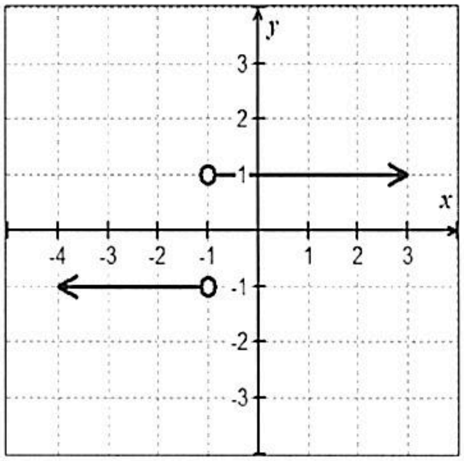
When finding limits, ask yourself, "What is happening to y as x gets close to a certain number?" You are finding the ***y*-value** for which the function is approaching as x approaches c.

LIMIT EXISTENCE THEOREM:

Verbally: The limit as x approaches c on $f(x)$ will exist if and only if the limit as x approaches c from the left is equal to the limit as x approaches c from the right.

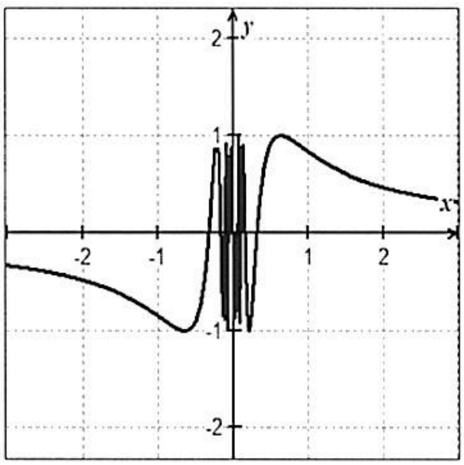
EX #3: Limits can fail to exist in three situations:

CASE 1: ____



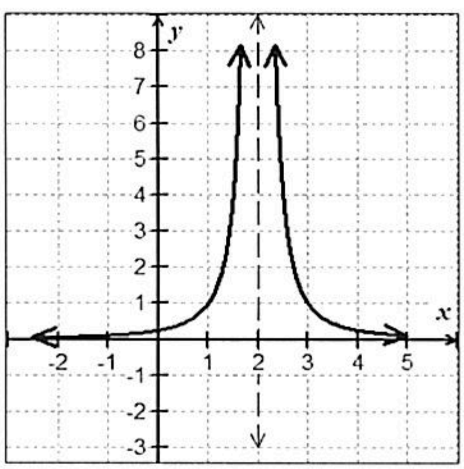
Justify why the limit does not exist at $x = -1$ for $f(x) = \frac{|x+1|}{x+1}$

CASE 2: ____



Justify why the limit does not exist at $x = 0$ for $f(x) = \sin\left(\frac{1}{x}\right)$

Case 3: ____



Justify why the limit does not exist at $x = 2$ for $f(x) = \frac{1}{(x-2)^2}$

EX #4: YOU OWN IT! In the box below, complete the sentence in your own words.

In order for the GENERAL LIMIT to exist, the function:

EX #5: Sketch a graph to satisfy each set of conditions.

1. $f(a)$ is undefined 2. $x = a$ is a point discontinuity 3. $\lim_{x \to a} f(x)$ exists	1. $\lim_{x \to a} f(x)$ DNE 2. $x = a$ is a jump discontinuity 3. $f(a)$ is undefined

EX #6: Finding limits from a table of values

Now, consider the function $f(x) = \frac{x-3}{x^2+2x-15}$. Complete the table below to find the limit as $x \to 3$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

Based on your analysis, what are the values of each of the limits below?

$\lim_{x \to 3^-} f(x) =$	$\lim_{x \to 3^+} f(x) =$	$\lim_{x \to 3} f(x) =$